Factorial Analysis of Al-Si-Mg Alloy Cast Hardness Based on Thickness and Ultimate Tensile Strength

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Abstract— Factorial analysis of the hardness of Al-Si-Mg alloy cast was carried out based on its thickness and ultimate tensile strength (UTS). A two-factorial empirical model was derived, validated and used for the analysis. The derived model showed that the hardness of the Al-Si-Mg alloy cast is a linear function of its thickness and UTS. The validity of the derived model expressed as: $\xi = 0.19\vartheta + 0.0003\Upsilon + 46.0$ was rooted on the model core expression $\xi - 0.0003\Upsilon = 0.19\vartheta + 46.0$ where both sides of the expression are correspondingly approximately equal. Evaluations from generated results indicated that the standard error incurred in predicting the hardness of Al-Si-Mg alloy cast for each value of its thickness & UTS considered, as obtained from experiment and derived model were 0.5497 and 4.8667×10^{-8}% & 0.4463 and 0.8758 % respectively. The hardness of Al-Si-Mg alloy cast per unit thickness & UTS as obtained from experiment and derived model results are 0.1857 and 0.19 HRB /mm & 0.2128 and 0.2177 HRB/ Nmm^{-2}, respectively and the correlations with thickness & UTS were all > 0.95. Deviational analysis shows that the maximum deviation of model-predicted hardness of Al-Si-Mg alloy cast from the experimental results is less than 2%. These invariably translated into over 98% operational confidence for the derived model as well as over 0.98 reliability response coefficients of Al-Si-Mg alloy cast hardness to its thickness & UTS.

Index Terms— Minimum 7 keywords are mandatory, Keywords should closely reflect the topic and should optimally characterize the paper. Use about four key words or phrases in alphabetical order, separated by commas.

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1 INTRODUCTION

The suitability of engineering materials for application in different service conditions involving wears and tears is largely dependent on their mechanical properties such as tensile strength, compressive strength, hardness, ductility etc. These properties are invariably controlled by the cooling rate of cast metals and alloys during solidification. The rate of cooling affects directly microstructural transformation during the solidification.

Conduction has been reported [1] to be the dominant mode of heat transfer from a solidifying cast. The report further revealed that conduction is the mechanism in which the heat is transferred internally within the solidifying metal and the mould.

Studies [2] have shown that convection involves the movement of the liquid metal during casting under the driving force of the density differences in the liquid. Results from the research show that it is a transport phenomenon where particles are carried by the fluid over some distances.

Convection has also been posited [3] as important because it affects the columnar to equiaxed transition. Similar research [4] has reported that convection driven by solutes can raise a number of problems for example heavy solutes cause the liquid to sink, and higher solutes cause floatation.

Research [5] has shown that the phenomenon of yielding occurs at the onset of plastic or permanent deformation; yield strength is determined by a strain offset method from the stress–strain behavior, which is indicative of the stress at which plastic deformation begins. The research [5] revealed that tensile strength corresponds to the maximum tensile stress that may be sustained by a specimen, whereas percents elongation and reduction in area are measures of ductility which is the amount of plastic deformation that has occurred at fracture. Resilience is the capacity of a material to absorb energy during elastic deformation; modulus of resilience is the area beneath the engineering stress–strain curve up to the yield point. Also, static toughness represents the energy absorbed during the fracture of a material, and is taken as the area under the entire engineering stress–strain curve. Ductile materials are normally tougher than brittle ones.

It has been reported [6] that hardness is a measure of the resistance to localized plastic deformation. In several popular hardness-testing techniques (Rockwell, Brinell, Knoop, and Vickers) a small indenter is forced into the surface of the material, and an index number is determined on the basis of the size or depth of the resulting indentation. For many metals, hardness and tensile strength are approximately proportional to each other.

The present work aims at carrying out a factorial analysis of the hardness of Al-Si-Mg alloy cast based on its thickness and UTS.

2 MATERIALS AND METHOD

The materials used for this research work includes aluminium scrap of 92.6% Al by composition (from First Aluminium Port Harcourt), silicon obtained from Metallurgical Training Institute, Onitsha, and Magnesium (in form lumps) obtained from Bridge Head Market Onitsha, Anambra state. The details of experiments carried out and the prevailing conditions are as stated in the report [7].

3 RESULTS AND DISCUSSIONS

As Table 1 shows that the hardness of Al-Si-Mg alloy cast are linearly dependent on its thickness and UTS.

Table1: Variation of Al-Si-Mg alloy cast hardness with its thickness and UTS [7]

(Υ)	(ϑ)	(ξ)
112.005	5	46.50
101.823	10	48.00
91.640	20	50.00
84.004	30	52.25
81.458	40	53.00

3.1 Model formulation

Computational analysis (using C-NIKBRAN: [8]) of results in Table 1 indicates that

$$\xi - K\Upsilon = N\vartheta + S \qquad (1)$$

Substituting the values of K, N and S into equation (2) reduces it to;

$$\xi - 0.0003\Upsilon = 0.19\vartheta + 46.0 \qquad (2)$$

$$\xi = 0.19\vartheta + 0.0003\Upsilon + 46.0 \qquad (3)$$

Where
K = 0.0003; N = 0.19 and S = 46.0 are equalizing constants (Determined using C-NIKBRAN [8])
(ξ) = Hardness (HRB)
(ϑ) = Thickness (mm)
(Υ) = UTS (Nmm^{-2})

3.2 Boundary and Initial Conditions

Considered ranges of hardness, UTS and Al-Si-Mg alloy cast thickness as 46.5 – 53.0, 18.458 – 112.005 Nmm^{-2}, and 5 – 40mm respectively.

MODEL VALIDATION

Table 2: Variation of $\xi - 0.0003\Upsilon$ with $0.19\vartheta + 46.0$

$\xi - 0.0003\Upsilon$	$0.19\vartheta + 46.0$
46.4664	46.95
47.9664	47.90
49.9664	49.80
52.2164	51.70
52.9664	53.60

The validity of the derived model was rooted in equation (2) where both sides of the equation are correspondingly approximately almost equal. Furthermore, equation (2) agrees with Table 2 following the values of $\xi - 0.0003\Upsilon$ and $0.19\vartheta + 46.0$ evaluated from Table 1.

Furthermore, the derived model was validated by comparing the model-predicted cooling temperature of Al-Si-Mg alloy cast and that obtained from the experiment. This was done using the 4th Degree Model Validity Test Techniques (4thDMVTT); statistical graphical, computational and deviational analysis.

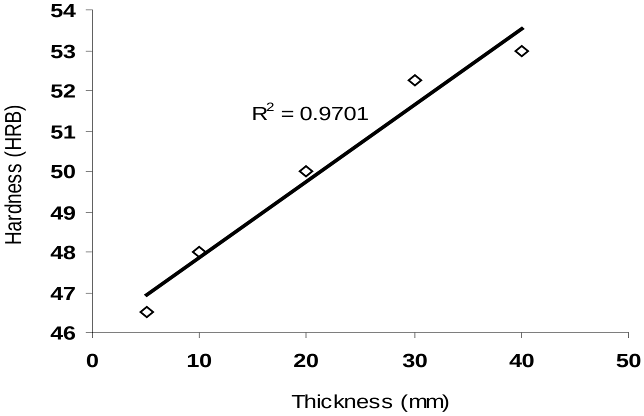

Fig. 1: Variation of Al-Si-Mg alloy cast hardness with its thickness as obtained from experiment [7]

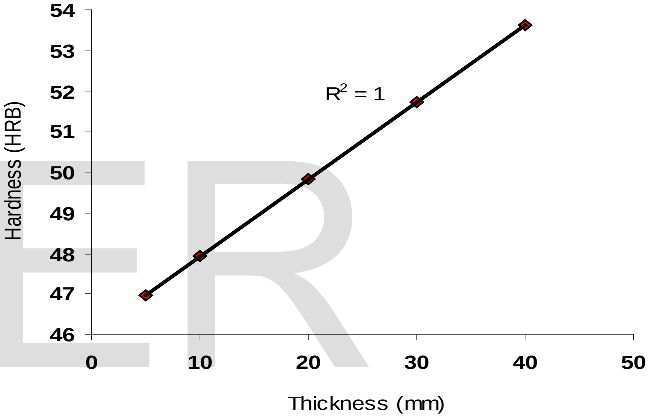

Fig. 2: Variation of Al-Si-Mg alloy cast hardness with its thickness as predicted by derived model.

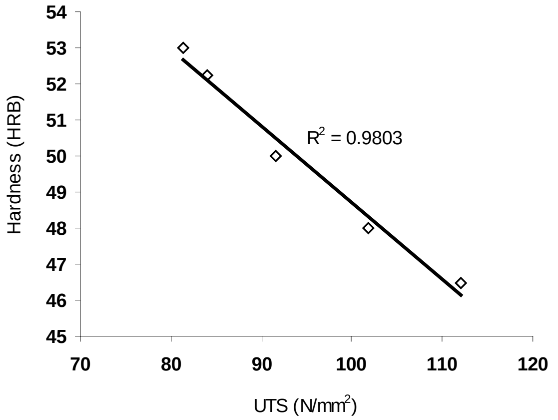

Fig. 3: Variation of Al-Si-Mg alloy cast hardness with its UTS as obtained from experiment [7]

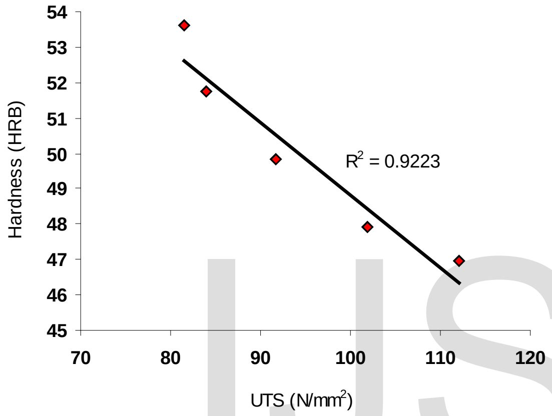

Fig. 4: Variation of Al-Si-Mg alloy cast hardness with its UTS as predicted by derived model.

3.3 Statistical Analysis

Standard Error (STEYX)

The standard errors incurred in predicting the hardness of Al-Si-Mg alloy cast for each value of the thickness & UTS considered as obtained from experiment and derived model were 0.5497 and 4.8667 x 10^{-4} & 0.4463 and 0.8758 % respectively. The standard error was evaluated using Microsoft Excel version 2003.

Correlation (CORREL)

The correlation coefficient between the hardness of Al-Si-Mg alloy cast and thickness & UTS were evaluated (using Microsoft Excel Version 2003) from results of the experiment and derived model. These evaluations were based on the coefficients of determination R^2 shown in Figs. 1- 4.

$$R = \sqrt{R^2} \qquad (4)$$

The evaluated correlations are shown in Tables 3 and 4. These evaluated results indicate that the derived model predictions are significantly reliable and hence valid considering its proximate agreement with results from actual experiment.

Table 3: Comparison of the correlations evaluated from derived model predicted and experimental results based on thickness

Analysis	Based on thickness	
	ExD	D-Model
CORREL	0.9849	1.0000

Table 4: Comparison of the correlations evaluated from derived model predicted and experimental results based on UTS

Analysis	Based on UTS	
	ExD	D-Model
CORREL	0.9901	0.9604

3.4 Graphical Analysis

Comparative graphical analysis of Figs. 5 and 6 show very close alignment of the curves from the experimental (ExD) and model-predicted (MoD) hardness of Al-Si-Mg alloy cast.

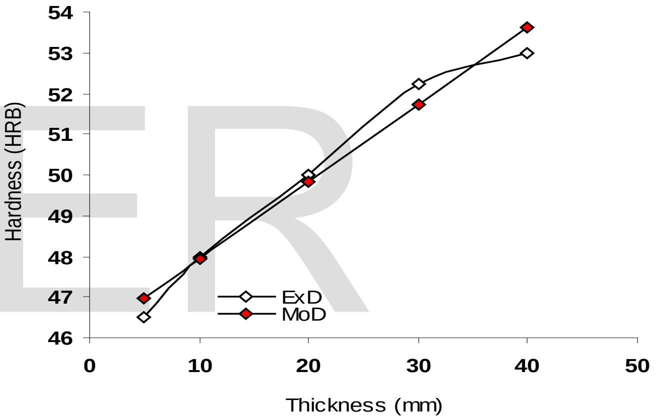

Fig. 5: Comparison of hardness values of Al-Si-Mg alloy cast (relative to the cast thickness) as obtained from experiment [7] and derived model.

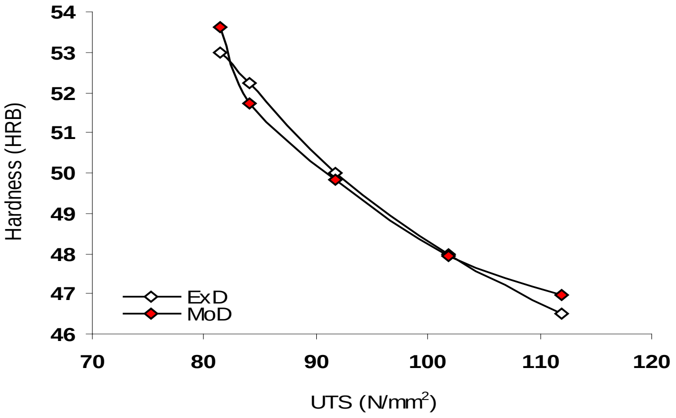

Fig.6: Comparison of hardness values of Al-Si-Mg alloy cast (relative to the cast UTS) as obtained from experiment [7] and derived model.

It is strongly believed that the degree of alignment of these curves is indicative of the proximate agreement between both experimental and model-predicted hardness values of the Al-Si-Mg alloy cast.

3.5 Computational Analysis

Smith Computational analysis of the experimental and model-predicted hardness of Al-Si-Mg alloy cast was carried out to ascertain the degree of validity of the derived model. This was done by comparing the hardness of Al-Si-Mg alloy cast per unit thickness and per unit UTS obtained from evaluation of experimental and model-predicted results.

Al-Si-Mg alloy cast hardness per unit thickness

The Al-Si-Mg alloy cast hardness per unit thickness was calculated from the expression;

$$\xi_t = \Delta\xi / \Delta\vartheta \qquad (5)$$

Equation (5) is detailed as

$$\xi_t = \xi_2 - \xi_1 / \vartheta_2 - \vartheta_1 \qquad (6)$$

$$\xi_t = \xi_2 - \xi_1 / \gamma_2 - \gamma_1 \qquad (7)$$

Where

$\Delta\xi$ = Change in the alloy hardness at two UTS and thickness values γ_1, γ_2, and ϑ_2, ϑ_1 respectively.

Considering the points (5, 46.5) & (40, 53) and (5, 46.98) & (40,53.63) as shown in Fig. 5, then designating them as (ζ_1, ϑ_1) & (ζ_2, ϑ_2) for experimental and model predicted results respectively, and also substituting them into equation (6), gives the slopes: 0.1857 and 0.19 HRB/ mm as their respective hardness of Al-Si-Mg alloy cast per unit thickness.

Al-Si-Mg alloy cast hardness per unit UTS

Also considering the points (112.005, 46.5) & (81.458, 53) and (112.005, 46.98) & (81.458, 53.63) as shown in Fig. 6, then designating them as (ζ_1, γ_1) & (ζ_2, γ_2) for experimental and model predicted results respectively, and also substituting them into equation (7), gives the slopes: - 0.2128 and - 0.2177 HRB/Nmm^{-2} as their respective hardness of Al-Si-Mg alloy cast per unit UTS.

It is very pertinent to state that the actual hardness of Al-Si-Mg alloy cast hardness per unit UTS (as obtained from experiment and derived model) was just the magnitude of the signed value. The associated sign preceding these values as evaluated signifies that the associated slope tilted to negative plane. Based on the foregoing, hardness of Al-Si-Mg alloy cast hardness per unit UTS as obtained from experimental and derived model predicted results are 0.2128 and 0.2177 HRB/Nmm^{-2}, respectively.

3.6 Deviational Analysis

Critical analysis of the Al-Si-Mg alloy cast hardness obtained from experiment and derived model show deviations on the part of the model-predicted values relative to values obtained from the experiment. This was attributed to the fact that the surface properties of the alloy cast and also the physico-chemical interactions between the matrix and alloying elements which played vital roles during processing were not considered during the model formulation. This necessitated the introduction of correction factor, to bring the model-predicted Al-Si-Mg alloys cast hardness to those of the corresponding experimental values.

The deviation Dv, of model-predicted Al-Si-Mg alloy cast hardness (from the corresponding experimental result) is given by

$$Dv = \frac{\xi_{MoD} - \xi_{ExD}}{\xi_{ExD}} \times 100 \qquad (8)$$

Where

ξ_{ExD} and ξ_{MoD} are Al-Si-Mg alloy cast hardness values obtained from experiment and derived model respectively.

Deviational analysis of Figs. 9 and 10 indicate that the precise maximum deviation of model-predicted Al-Si-Mg alloy cast hardness from the experimental results is less than 2%. This invariably translated into over 98% operational confidence for the derived model as well as over 0.98 reliability response coefficients of Al-Si-Mg alloy cast hardness to its thickness & UTS.

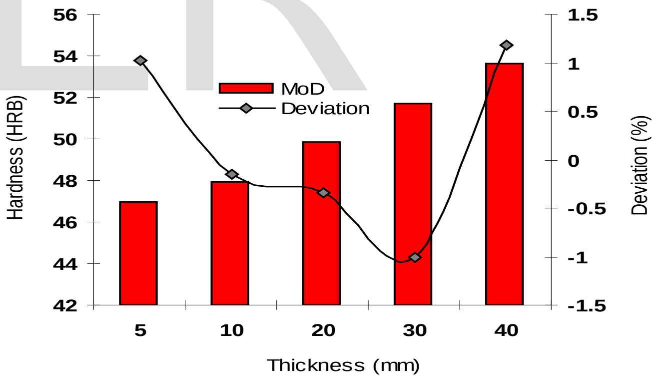

Fig. 9: Variation of model-predicted hardness of Al-Si-Mg alloy cast (relative to thickness) with associated deviation from experiment.

Consideration of equation (8) and critical analysis of Figs. 9 and 10 show that the least and highest magnitudes of deviation of the model-predicted hardness of Al-Si-Mg alloy cast (from the corresponding experimental values) are - 0.15 and + 1.19%. Figs. 9 and 10 indicate that these deviations correspond to Al-Si-Mg alloy cast hardness values: 47.93 and 53.63 HRB, thicknesses: 10 and 40 mm as well as UTS values: 101.823 and 81.458 Nmm^{-2} respectively.

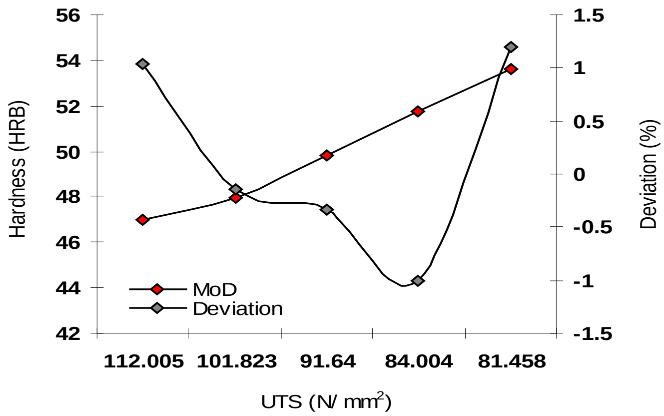

Fig. 10: Variation of model-predicted hardness of Al-Si-Mg alloy cast (relative to UTS) with associated deviation from experiment.

Correction factor, Cf to the model-predicted results is given by

$$Cf = - \left(\frac{\xi_{MoD} - \xi_{ExD}}{\xi_{ExD}} \times 100 \right) \qquad (9)$$

Critical analysis of Figs. 9, 10 and Table 5 indicates that the evaluated correction factors are negative of the deviation as shown in equations (8) and (9).

Table 5: Variation of correction factor (to model-predicted hardness of Al-Si-Mg alloy cast) with thickness and UTS

(γ)	(ϑ)	Correction factor (%)
112.005	5	- 1.03
101.823	10	+ 0.15
91.640	20	+ 0.34
84.004	30	+ 1.00
81.458	40	- 1.19

Table 5 shows that the least and highest correction factor (to the model-predicted hardness of Al-Si-Mg alloy cast) are + 0.15 and – 1.19%.

Since correction factor is the negative of deviation as shown in equations (8) and (9), Table 5, Figs. 9 and 10 indicate that these highlighted correction factors correspond to Al-Si-Mg alloy cast hardness values: 47.93 and 53.63 HRB, thicknesses: 10 and 40 mm as well as UTS values: 101.823 and 81.458 Nmm^{-2} respectively.

The correction factor took care of the negligence of operational contributions of the surface properties of the alloy cast and also the physico-chemical interactions between the matrix and alloying elements which actually played vital role during the cooling process. The model predicted results deviated from those of the experiment because these contributions were not considered during the model formulation. Introduction of the corresponding values of Cf from equation (9) into the model gives exactly the corresponding experimental values of the Al-Si-Mg alloy cast hardness.

It is very pertinent to state that the deviation of model predicted results from that of the experiment is just the magnitude of the value. The associated sign preceding the value signifies that the deviation is a deficit (negative sign) or surplus (positive sign).

5. CONCLUSION

Factorial analysis of the hardness of Al-Si-Mg alloy cast was carried out based on its thickness and ultimate tensile strength (UTS). A two-factorial empirical model derived and validated was used for the analysis. The derived model showed that the hardness of the Al-Si-Mg alloy cast is a linear function of its thickness and UTS. The validity of the derived model was rooted on the model core expression $\xi - 0.0003\gamma = 0.19\vartheta + 46.0$ where both sides of the expression are correspondingly approximately equal. Evaluations from generated results indicated that the standard error incurred in predicting the hardness of Al-Si-Mg alloy cast for each value of its thickness & UTS considered, as obtained from experiment and derived model were 0.5497 and 4.8667×10^{-8}% & 0.4463 and 0.8758 % respectively. The hardness of Al-Si-Mg alloy cast per unit thickness & UTS as obtained from experiment and derived model results are 0.1857 and 19.0 HRB /mm & 0.2128 and 0.2177 HRB/ Nmm^{-2}, respectively and the correlations with thickness & UTS were all > 0.95. Deviational analysis shows that the maximum deviation of model-predicted hardness of Al-Si-Mg alloy cast from the experimental results is less than 2%. These invariably translated into over 98% operational confidence for the derived model as well as over 0.98 reliability response coefficients of Al-Si-Mg alloy cast hardness to its thickness & UTS.

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